

1. Let G be a graph. Give the definition of the following terms. (One point each.)
 - (a) Vertex cover number of G .
 - (b) Independence number of G .
 - (c) Domination number of G .
 - (d) Chromatic number of G .
 - (e) Crossing number of G .
2. Let $G = \langle X, Y \rangle$ be a bipartite graph. Prove that G has a matching that saturates X if and only if $|N_G(S)| \geq |S|$ for all subsets S of X . (Hall's Theorem)
3. (a) Prove that if G is a ^{connected} planar graph with p vertices, q edges and f faces, then $p - q + f = 2$. (2 points)
 (b) Use (a) to prove that if a ^{connected} (p, q) -pseudograph G has a 2-cell embedding on the surface of genus n with f faces, then $p - q + f = 2 - 2n$. (3 points)
4. Let P denote the Petersen graph. Prove that (a) P is of Class 2 (3 points) and P does not contain a Hamilton cycle (2 points).
5. Find the Ramsey number $R(3, 4)$. (Explain your answer in details.)
6. (a) Construct a 2-connected graph G such that $\text{diam}(G) = 10$ and $\text{rad}(G) = 7$.
 (b) Construct a graph G such that $\kappa(G) = 5$ and $\kappa'(G) = 8$.
 (Each one has 2.5 points.)
7. (a) State the max-flow min-cut theorem. (2 points)
 (b) Use (a) to find the maximum flow value of the following network.

