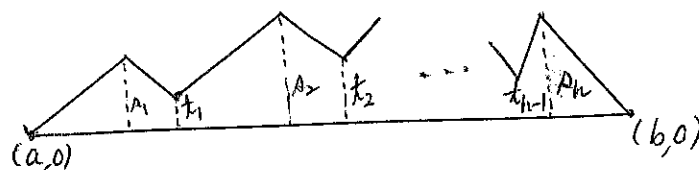


1. A mountain range is a polygonal curve from  $(a, 0)$  to  $(b, 0)$  in the upper half-plane. Hikers A and B begin at  $(a, 0)$  and  $(b, 0)$  respectively.
- (i) *Prove that A and B can meet by traveling on the mountain range in such a way that at all times their heights above the horizontal axis are the same.* (ii) Let  $\langle 0, s_1, t_1, s_2, t_2, \dots, t_{h-1}, s_h, 0 \rangle$  denote the sequence of the heights depicted in the following figure. Determine where is the meeting point according to the given sequence.



2. (a) List all non-isomorphic graphs of order 5. (Explain your answer.)  
 (b) Give an idea of determining whether two given graphs are isomorphic or not.
3. Find  $ex(10; C_4)$  (2 points) and  $ex(91; C_4)$  (3 points).  
 Bonus: Find  $ex(n; C_4)$  for other positive integer  $n$ ?
4. For positive integers  $a$  and  $b$ ,  $a \leq b \leq 2a$ , construct a 2-connected graph  $G$  such that  $\text{diam}(G) = b$  and  $\text{rad}(G) = a$ . Work on  $a = 7$  and  $b = 11$  for 2 points.  
 Bonus: How about  $G$  with larger connectivity?
5. For positive integers  $a$ ,  $b$  and  $c$ ,  $a \leq b \leq c$ , find a graph  $G$  such that  
 $\kappa(G) = a$ ,  $\kappa'(G) = b$  and  $\delta(G) = c$ .

Bonus problem

Find as many  $(d, g)$ -cages as possible and prove that these cages are 2-connected.