

1. Find  $R(3, 4)$  and  $R(4, 4)$ . (5 points)
2. Let  $G$  be a  $(p, q)$ -pseudograph which has a 2-cell embedding on  $S_n$ . Prove that  $p - q + f = 2 - 2n$  where  $f$  is the number of faces in the embedding. (5 points)
3. Let  $G$  be a connected graph which is neither a complete graph nor an odd cycle. Prove that the chromatic number of  $G$ ,  $\chi(G) < \Delta(G) + 1$ . (5 points)
4. Prove that in an  $n$ -connected graph  $G$ , any  $n$  distinct vertices are contained in a cycle of  $G$  for  $n > 2$ . (5 points)
5. Prove that in a bipartite graph  $G$  defined on  $(X, Y)$ ,  $G$  has a matching saturated  $X$  if and only if  $|N(S)| \geq |S|$  for all  $S \subseteq X$ . (5 points)
6. Prove that Petersen graph can't be decomposed into three 1-factors and Petersen graph does not contain a Hamilton cycle. (5 points)
7. Find  $z(91, 91; 2, 2)$ , that is the maximum number of edges of a subgraph of  $K_{91, 91}$  which does not contain  $K_{2, 2}$ . (5 points).

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