Graph Theory Third Exercise Due 2024, 12, 18

1. Prove that $G(n; \{1, 2\})$ can be decomposed into a prescribed 2-factor and a Hamilton cycle. (The prescribed 2-factor can be any 2-factor of order n.) (5 points)

2. Find R(3, 4), R(4, 4) and R(3, 3, 3). (5 points) Bonus for extra works.

3. Let G be a (p, q)-pseudograph which has a 2-cell embedding on S_n. Prove that $p - q + f = 2 - 2n$ where f is the number of faces in the embedding. (5 points)

4. Find the crossing number of K_8 and $K_{4,4}$ respectively. (5 points)

5. Let G be a connected graph which is neither a complete graph nor an odd cycle. Prove that the chromatic number of G, $\chi(G) \leqq \Delta(G)$. (5 points)

6. What is the discharging method? Give two examples to describe the idea. (5 points)

7. Bonus: A Class 2 cubic graph is called a snark. Find as many snarks as possible. (At most 5 points)