

1. Find at least 5 different cages which are not trivial. (5 points)
2. Show that a bipartite graph G with 16 edges contains an induced subgraph G' with 8 edges. (5 points)
How about the cases when G has 32 edges or 64 edges? (Bonus)
3. Prove that in an n -connected graph G , any n distinct vertices are contained in a cycle of G . ($n = 3$ for 2 points and $n > 3$ for 3 points.)
4. Find a second proof of Menger's Theorem which is different from the one given in Lecture 6. (5 points)
5. Find at least three different ways to prove Hall's Theorem. (5 points)
6. Prove that Petersen graph can not be decomposed into three 1-factors and Petersen graph does not contain a Hamilton cycle. (5 points)
7. (Bonus) Let G be the circulant graph $G(n; \{1,2\})$. Find the independence number of G . (Worth at most 5 points.)