

Investigating Combinatorics (學習組合學)

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What is Combinatorics?

- Combinatorics is a branch of mathematics concerning the study of finite or countable structures.
- Well-known fields: Graph theory, Design Theory, Enumerative Combinatorics, Algebraic Combinatorics, Algorithms and Combinatorial Optimization.
- Others: Coding Theory, Cryptography, Networks and Group Testing.



包羅萬象

只要是可算(數)的都是學習的對象。



Are you sure?

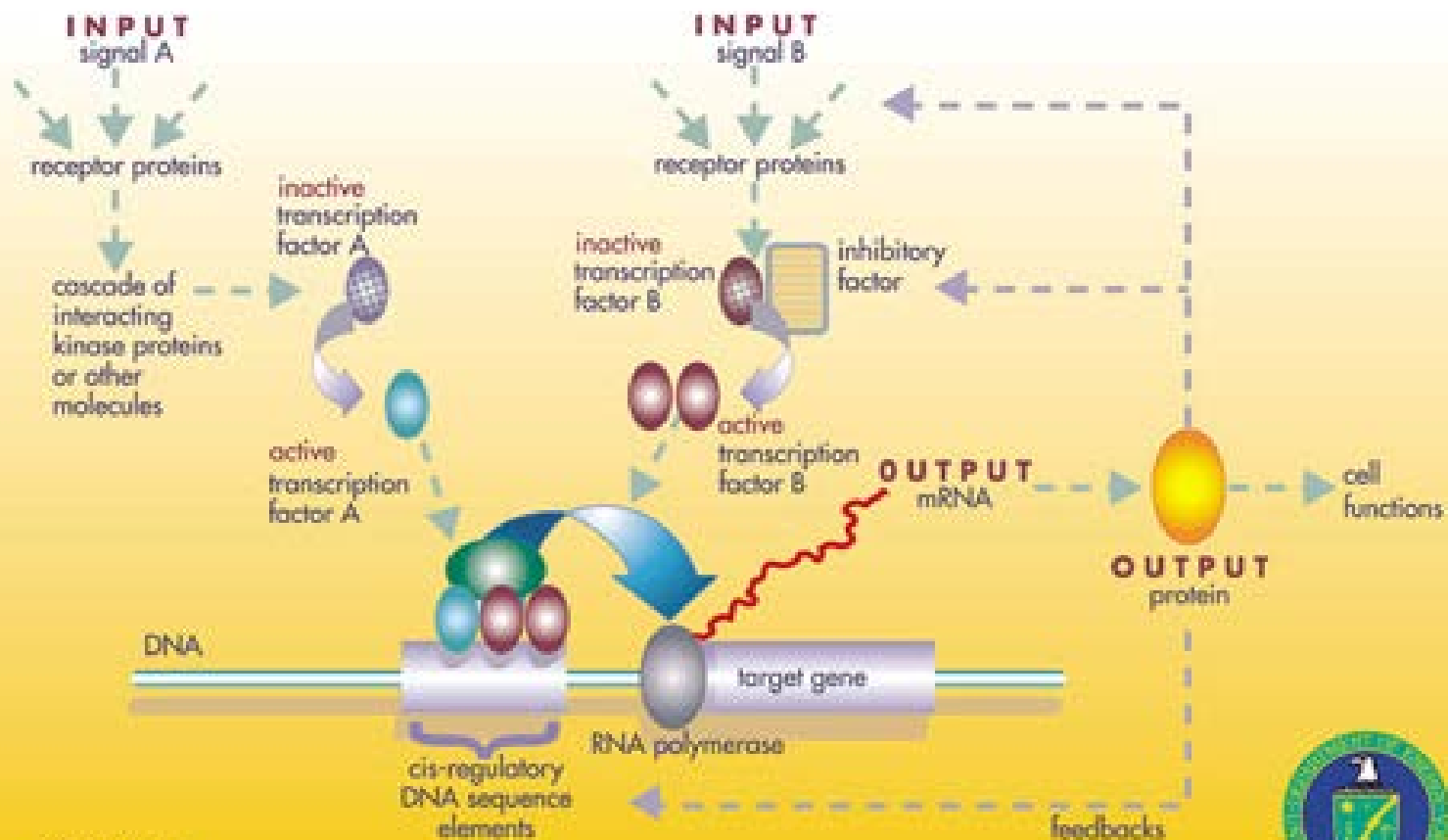


優先選擇圖論

- In mathematics **graph theory** is the study of graphs, which are mathematical structures used to model pairwise relations between objects.
- In fact, newly developed idea, called *hypergraph*, shows that we can model mutual relations of more than two objects.
- Even in Social Science, we have the so-called Social Network. It plays an important role in modern society.

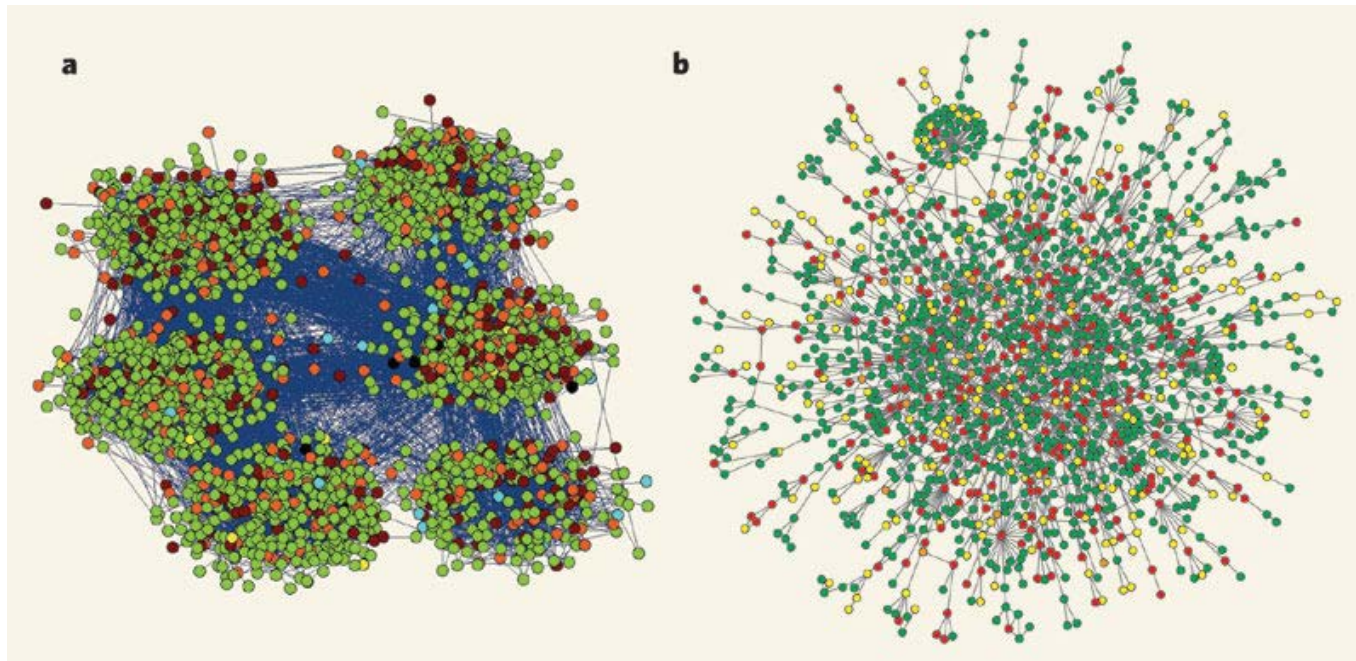


A GENE REGULATORY NETWORK



Applications of Graph Theory

- They are everywhere.
- With colorings, graphs are beautiful.
- Digraphs are applicable to many aspects of our life. (Complex networks)



Most Interests

- Graph coloring (Various versions)
- Graph labelling
- Graph decomposition
- Domination problem (Various types)
- Hamiltonian problem
- Independent sets
- De-cycling numbers
- Networks
- Extremal graphs
- Many topics!



Contributions

- I believe, in recent years, Chinese graph theorists play the most important role in the development of this topic.
- India graph theorists also contribute a lot.
- Taiwan researchers do have their influence on both of them.



Probabilistic method

- A non-constructive method, primarily used in **combinatorics** and pioneered by **Paul Erdős**, for proving the existence of a prescribed kind of mathematical object: **Graphs**.
- It works by showing that if one randomly chooses objects from a special class, **the probability that the result is of the prescribed kind is more than zero**.
- Although the proof uses probability, the final conclusion is determined for *certain*, without any possible error.
- **人際關係圖: Diameter 6!**



IT DOES EXIST!

- Who knows what it looks like.
- But, it exists.
- This is not only for talking about random graphs or random walks.
- Random is great!



演算法

- How fast can we obtain the answer?
- Does many mean more than one?
- We can not live without an **algorithm**!
- Simply, we are asking ourselves: what's next?

What is algorithm?

- A method that can be used by a computer for the solution of a problem.
- A sequence of computational steps that transform the input into the output.
- Examples of algorithms in the nature: DNA and cook book.
- The word "algorithm" comes from the name of [Abu Ja'far Mohammed ibn Musa al Khwarizmi](#) (780–825 A.D.), an Arab mathematician who wrote a textbook on mathematics.



Data Analyst

- The hottest topic in recent years is called “**Data Science**”. Simply, for practical reason, we need more information from each individual and each group as well.
- Using correct data becomes an important issue everywhere in the world. (無人車, 無人飛機)
- But, how?
- **Combinatorics!** 組合學!
- *Finite: Countable.*



Enumerative combinatorics

- For those who are interested in Data Analysis, they should not forget to learn more combinatorics, it is going to play an important role.
- More Algebra is get involved in this research.



Combinatorial Designs (組合設計)

- **Combinatorial design theory** is the part of combinatorial mathematics that deals with the existence, construction and properties of systems of finite sets whose arrangements satisfy generalized concepts of *balance and/or symmetry*.
- Combinatorial design theory can be applied to the area of design of experiments. Modern applications are also found in a wide gamut of areas including; Finite geometry, tournament scheduling, lotteries, mathematical biology, algorithm design and analysis, networking, coding theory, group testing and cryptography.



As evenly as possible

- This takes care of constructing a good experimental **design**. (**Combinatorial Design**)
- Also, it provides a chance to construct an optimal **code** in **Coding Theory**.
- In **cryptography**, it also plays an important role in many topics such as **Secret Sharing Schemes**!
- Balanced designs are candidates of optimal designs which is important in **Statistics**.
- Good codes from designs are useful in **Communication Network** for sending packets.



GREAT DESIGN!



GLOBAL PACKING NUMBER

- Given $\mathcal{P} \in \mathfrak{B}_{G,N}$.
- A **k -global packing** of $(G, N, \mathcal{P})$ is a mapping $\phi : \mathcal{P} \rightarrow \{1, 2, \dots, k\}$, such that $\phi(P) \neq \phi(P')$ if $P, P' \in \mathcal{P}$ and P, P' have one or more arrows in common.
- $\Phi(G, N, \mathcal{P})$: the smallest k that allows a k -global packing of $(G, N, \mathcal{P})$.

Definition

The global packing number of (G, N) is

$$\Phi(G, N) := \min_{\mathcal{P} \in \mathfrak{B}_{G,N}} \Phi(G, N, \mathcal{P}).$$

- $\mathcal{P}$ is said to be **ideal** if $\Phi(G, N) = \Phi(G, N, \mathcal{P})$.



Definitions

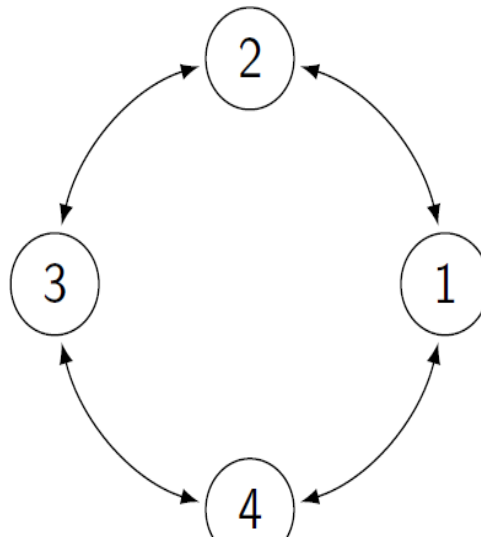
Definition

A path system of G with respect to N is a set of directed paths

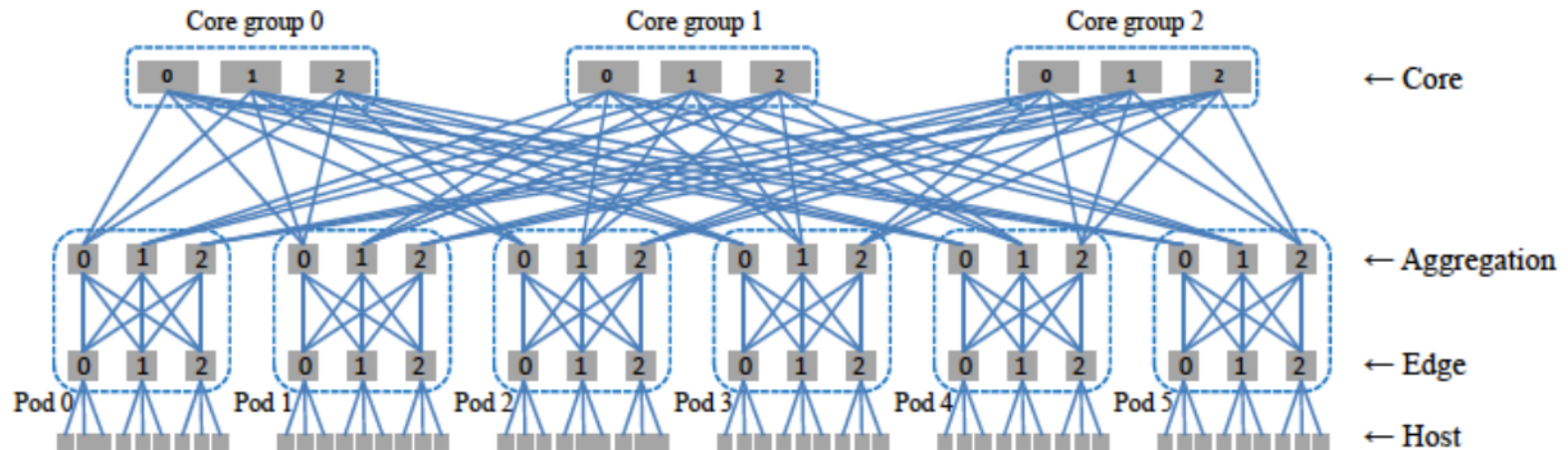
$$\mathcal{P} := \{P_{(a,b)} : a, b \in N, a \neq b\},$$

one for each distinct ordered node pair.

- $\mathfrak{B}_{G,N}$: the set of all path systems of (G, N) .



Fat-tree network



- $\mathbf{T}_n$: n -ary fat-tree network. Each switch has $2n$ ports and the number of pods is $2n$.
- $c_{i,j}$: the j -th core switch in the i -th core group. $|C_n| = n^2$
- $a_{i,j}$: the j -th aggregation switch in the i -th pod. $|A_n| = 2n^2$.
- $e_{i,j}$: the j -th edge switch in the i -th pod. $|E_n| = 2n^2$.
- $h_{t,i,j}$: the j -th host under the edge switch $e_{t,i}$. $|H_n| = 2n^3$.

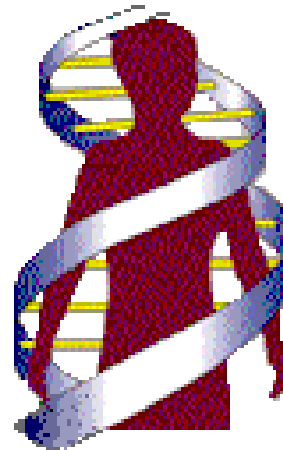
COMMUNICATION PROBLEM

- The answers combine the effort of using graph theory and combinatorial designs, especially graph colorings and Latin squares.
- This approach also works in constructing various kinds of sequences which are applicable in communications such as User irrepressible sequences and Conflict-avoiding codes.
- Rumor Centrality problem is another important application.



Positive or negative

- A simple idea: Just ask!
- How to ask?
- What can we get from the answers?
- **Group testing** is a wonderful tool in modern era.
- Especially in DNA sequencing!



Group Testing

- This idea is very powerful in many applications not only on some special *algorithmic* topics.
- Despite of using computer, we also need a *smarter mind*!
- So, what is it? You should have known this notion in this department!
- Two leading experts in the world are Chinese:
堵丁柱及黃光明。
- They have written two books to promote the research of this topic.



GROUP TESTING WORKS!

- In screening clone library the goal is to determine which clones in the clone library hybridize with a given probe in an efficient fashion. A clone is said to be **positive** if it hybridize with the probe(探針), and **negative** otherwise.
- Our goal is to provide an experimental protocol that **identifies all pairs of adjacent primers** with as few PCRs (queries) (or multiplex PCRs respectively) as possible.
- There are others: **finding the pattern of data!**
- **Ask Prof. Huilan Chang for more details!**



HAVE YOU SEEN SOMETHING DIFFERENT?



FINGERPRINTS IN MULTI-MEDIA

- We may water-mark several pixels of a picture by using colors which are very closed to nearby pixels; colors are represented by binary vectors.
- Each buyer has his or her own fingerprints in order to make sure they have the right to see the picture.
- Any one who did not pay and used the picture will be recognized right away.
- The idea comes from the construction of **identifiable parent property codes (IPP)**.



My journey on combinatorics

- I was lucky to know this term “combinatorics” when I was an undergraduate student.
- It was in the summer of 1971 (I remembered). At that year Professor Frank K. Hwang (黃光明教授) gave a short course of this topic at NTU(台大).
- The book he used was “An Introduction to Combinatorial Theory” by M. Hall. (1st edition)
- This topic was fairly new at that time. Most of us in the class had not heard this “strange” term before.



Coincidence

- I chose a university in the States to learn **Topology** which I like very much.
- I was the only student to choose that course in Fall, 1977 and therefore it was cancelled due to the choice of lecturer. (He needs to teach one extra course if he decides to do it.)
- So, besides the regular selection of courses such as Analysis and Algebra, I took “**Combinatorics**” lectured by **Curt C. Lindner** who became my supervisor of theses, both M.S. and Ph. D.
- In my first lesson I learned “**Latin squares**” and I ended my study at Auburn (Dec. 1980) with a thesis related to this notion.



My investment

- I did (do) not have much money to worry about over past years.
- So, I invested **my time** in investigating **combinatorics!**
- With the investment on combinatorics, I am able to tell you: **it is worth of doing it for a long while.**
- Not only on application part but also in research, there are rooms for improvement.
- **We count on you!!!**



Your investment

- Money or Time?

Investing your **time** if you do not have money!

- Mathematics or Others?

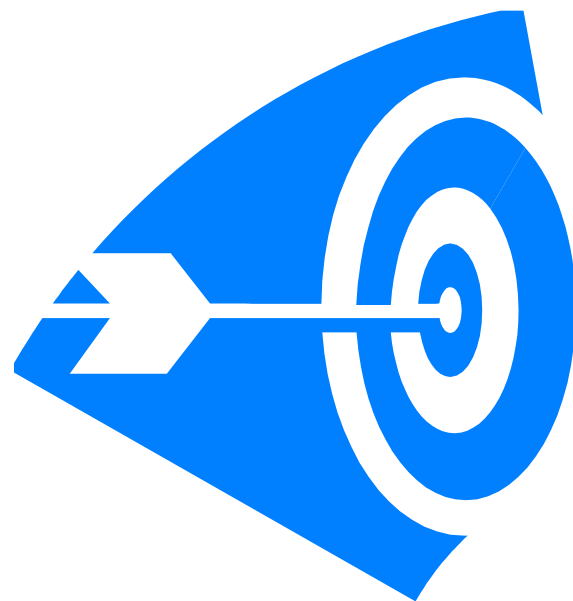
Learning Math. gets you smarter (hopefully)!

- Combinatorics or other fields in Mathematics?

This world is full of Combinatorics!



○進可攻→退可守



加油！

Keep Moving Forward!

