Definition 4

1. Topological graph theory

   The primitive objective of **topological graph theory** is to draw a graph on a surface so that no two edges cross, an intuitive geometric problem that can be enriched by specifying symmetries or combinatorial side-conditions.

2. 2-manifold

   A **2-manifold** is a connected topological space in which every point has a neighborhood homeomorphic to the open unit disk defined on $\mathbb{R}^2$.

3. n-manifold

   An **n-manifold** is a connected topological space in which every point has a neighborhood homeomorphic to the n-dimension unit ball $B_n = \{(x_1, x_2, ..., x_n) \in \mathbb{R}^n | \sum_{i=1}^{n} x_i^2 < 1\}$.

4. Bounded

   A surface M of $\mathbb{R}^3$ is **bounded** if there exists a positive real number K such that $M \subseteq \{(x, y, z) | x^2 + y^2 + z^2 \leq K\}$.

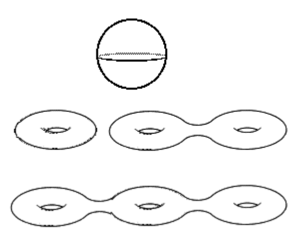
5. Closed

   Let $M \subseteq \mathbb{R}^3$ be a 2-manifold. Then M is said to be **closed** if it is bounded and the boundary of M coincides with M.

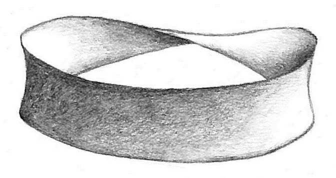
6. Orientable and Non-orientable surfaces

   There are two kinds of closed surfaces, **orientable** and **non-orientable**. Let $M(\subseteq \mathbb{R}^3)$ be a 2-manifold; M is said to be **orientable** if for every simple closed C on M, a clockwise sense of rotation is preserved by traveling once around C. Otherwise, M is **non-orientable**.

   - The sphere, the torus, the double torus, the triple torus, and so on, they are orientable. They are commonly denoted S_0, S_1, S_2, S_3,... .



- The Möbius band is a surface that is neither closed nor orientable.



7. Imbeddings

   To formalize the notion of a drawing without crossings, we define an **imbedding** of a graph in a surface to be a continuous one-to-one function from a topological representation of the graph into the surface.

8. Isomorphism

   A graph map $G \to G'$ is called an **isomorphism** if both its vertex function and its edge function are one-to-one and onto.

9. Homeomorphic

   The graph G and H are called **homeomorphic** if they have respective subdivisions G' and H' such that G' and H' are isomorphic graphs.

10. Regions(or Faces)

    If a connected graph is imbedding in a sphere, then the complement of its image is a family of **regions**(or **faces**), each homeomorphic to an open disk. In more complicated surfaces, the regions need not be open disks.

    If it happens that they are all open disks, then the imbedding is called a **2-cell(or cellular) imbedding**.

11. Genus

    If a surface is orientable, its **genus** is defined to be the number of handles one must add to the sphere to obtain its homeomorphism type.