Week 17

June 15th

Review

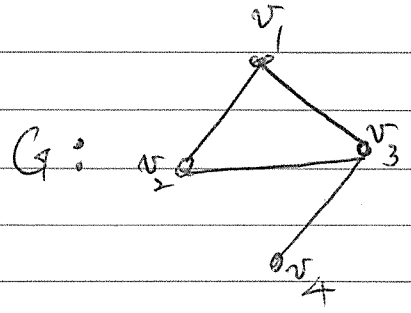
Theorem If A is a real symmetric matrix, then all its eigenvalues are real numbers.

(*) We may then use $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ to denote the n eigenvalues of A. $\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ is the spectrum of A and λ_1 is called the spectra radius of A.

An important application of the above theorem can be found in Algebraic Graph Theory.

Definition Let G be a graph with n vertices $v_1, v_2, \cdots, v_n$. $A(G) = [a_{ij}]$ is an adjacency matrix of G where $a_{ij} = 1$ provided v_i is adjacent to v_j and $a_{ij} = 0$ otherwise.

Example



$$A(G) = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Now, we can use what we have learned this semester to find $\lambda_1, \lambda_2, \lambda_3$ and λ_4 and we know all of them are real numbers since $A(G)$ is a real symmetric matrix.

(**) The main problem we study in Algebraic Graph Theory is to "determine" the structure of a graph G by using the eigenvalues of $A(G)$.

(***) It is known that there are non-isomorphic graphs which have the same spectrum. These graphs are called <u>cospectra</u>.

Review for the 3rd test. . . .

Good Luck !