Linear Algebra (II) Test (II)

2011,05,11

1. Let $A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

(a) Find all eigenvalues of A.(8 points)

(b) Find $E_A(\lambda)$ for each eigenvalue λ of A.(8 points)

(c) Find the algebraic multiplicity and geometric multiplicity of each eigenvalue of A. (6 points)

(d) Find a matrix P and a diagonal matrix D such that $A = PDP^{-1}$. (8 points)

(e) Find A^9. (8 points)

2. Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 5 & 6 \\ 2 & -4 & 5 \end{bmatrix}$.

Use Cayley-Hamilton Theorem to find A^{-1}. (8 points)

3. Find a matrix with trace 1, determinant -2, and two eigenvectors $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. (12 points)

4. Are the following matrices diagonalizable? (Explain your answer.)

$(a) \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (b) \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$. (16 points)

5. Find $A^{200}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ where $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. (16 points)

6. Prove that an $n \times n$ matrix A is diagonalizable if and only if it has n independent eigenvectors. (20 points)